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Three-Dimensional Response Characteristics for Spacecraft with Deploying Flexible Appendages

K.W. Lips* and V.J. Modi†
University of British Columbia, Vancouver, B.C., Canada

This paper represents a wide-ranging parametric evaluation of the general three-axis librational response of a large class of satellites with deploying beam-type flexible appendages. The effect of system parameters such as deployment rate, appendage offset, center-of-mass shift, satellite inertias, etc., are studied for both transient and steady-state performance. The results clearly establish a significant influence of flexibility on attitude motion during appendage deployment. They show the three-axis response to be markedly different from the simplified planar data, which often failed to predict instability. On the other hand, in general, offset and shifting center of mass were found to have an insignificant effect on the response for the cases considered. This may permit considerable simplification of the governing nonlinear, nonautonomous, coupled, hybrid differential equations of motion with associated saving in the computational time and effort.

I. Introduction

FLEXIBILITY effects on satellite attitude motion and its control continues to be a subject of considerable importance. This problem tends to become more critical with the growth in size of flexible members such as booms, solar panels, antennae, etc. Over the years, a large body of literature pertaining to the various aspects of satellite system response, stability, and control has evolved, which is effectively reviewed in Refs. 1-13.

Flexibility is a design choice dictated in part by a dichotomy of extremes in the force environment: very high accelerations during delivery to orbit followed by very low accelerations in the operational life. Structures having large dimensions are often required to conduct experiments, provide stabilization, and generate power. However, configuration size and weight are often severely constrained as a result of the launch vehicle limitations or structural strength of the satellite components. As a solution, spacecraft are initially packaged as compact rigid bodies. Once in orbit, various elements deploy to establish the desired configuration. Deployment, in many instances, accompanies the attitude acquisition phase during which large-angle maneuvers take place. The presence of environmental forces such as solar radiation pressure, the Earth's magnetic field, free molecular forces, etc., which are capable of exciting elastic degrees of freedom, add to the complication.

With an increasing use of flexible appendages, the problem grows more critical as stationkeeping and pointing requirements become stringent. For example, the Canada/U.S.A. Communications Technology Satellite (CTS/Hermes), launched in 1976, carried two solar panels, 1.1×7.3 m each, to generate 1.2 kW. The "Galileo," scheduled for a 1982 launch, has articulated members with both spinning and nonspinning sections making up the main body. Attached to the spinning part are flexible booms up to 11 m in length. In addition, wide variations are expected to occur in the inertia properties over the life of the mission, due to a relatively large ratio of propellant to spacecraft mass. Structural members of the proposed solar power satellites will have dimensions measurable in kilometers. With the advent of the Space Shuttle, tether-supported satellite systems extending to 100 km are anticipated.

It is well recognized that prediction of satellite attitude motion is by no means a simple proposition, even if the system is rigid. The timebound character of most projects restricts attention to a given configuration with dynamic simulation confined to phases considered most important. It is, therefore, understandable why the majority of published papers discuss only briefly steady-state motion. Transient behavior associated with the critical phase-of-attitude acquisition and/or deployment-related maneuvers has been largely ignored. It should be mentioned that deployment effects, although of a transient nature, may be felt over a long period of time as a result of relatively small extension rates that can be associated with long appendages. For example, extension of a 200-m boom may require around 2000 s. In addition, deployment affects the force field acting on the flexible members, thus influencing elastic responses, structural integrity, and the libration itself.

This paper studies librational dynamics of spacecraft in an eccentric orbit with an arbitrary number, type, and orientation of deploying flexible appendages. The governing nonlinear, nonautonomous, and coupled equations of motion are integrated using a digital computer. The effect of important system parameters is assessed through illustrative configurations representing a large class of gravity-gradient and spinning satellites. Rather than accumulation of a large amount of data, the emphasis is on evolution of a generalized and organized methodology for coping with such complex dynamical systems.

II. Formulation of the Problem and Numerical Analysis

A. Attitude Dynamics

Consider a spacecraft, consisting of an arbitrarily shaped central rigid body and flexible deploying appendages, with its instantaneous center of mass at O_c in an arbitrary orbit around the center of force at O_I (Fig. 1a). Elastic appendages of arbitrary shape and orientation are cantilevered to the central rigid body forming a simplified topological tree. The system is free to undergo three-dimensional vibrational and librational motions in the gravity-gradient field. Position vector $\boldsymbol{R}_c$ and true anomaly θ define the position of O_c with respect to the inertial reference coordinates X,Y,Z. X_0, Y_0,Z_0 represent an orthogonal orbiting reference frame with

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*Graduate Research Assistant, Dept. of Mechanical Engineering. Member AIAA.
†Professor, Dept. of Mechanical Engineering. Member AIAA.

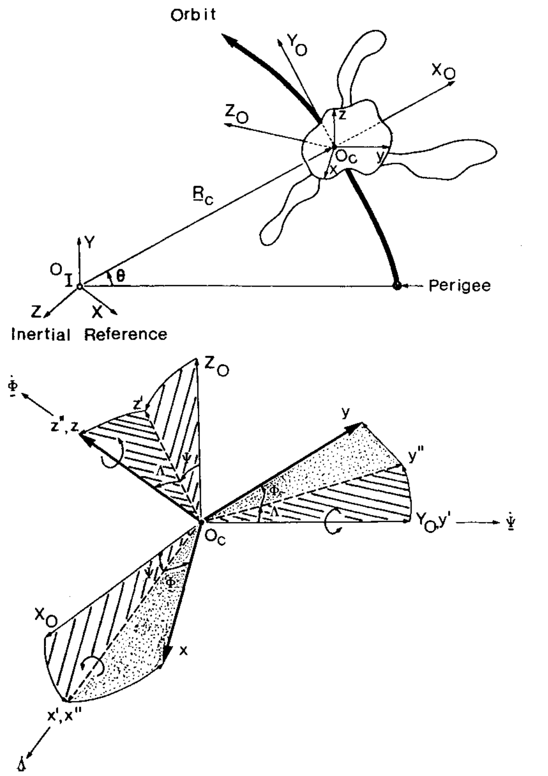

Fig. 1 Geometry of satellite motion: a) inertial, rotating, and body-fixed coordinate systems; b) modified Eulerian rotations—Ψ,Λ,Φ defining arbitrary orientation of the central rigid body during librations.

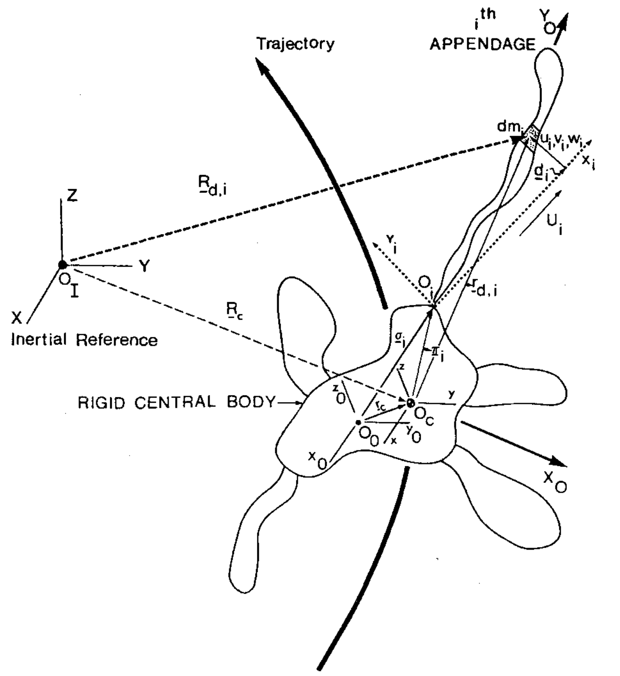

Fig. 2 Geometry of general spacecraft configuration accounting for a shift in center of mass, appendage offset, deployment, and deformation.

its origin at O_c, where X_0 and Y_0 are local vertical and horizontal, respectively, and Z_0 is parallel to orbit normal. Librational response of the central body is defined by the Eulerian rotations ψ, Λ, and Φ (roll, yaw, and pitch, respectively) of the body-fixed axes x,y,z with respect to the orbiting reference X_0,Y_0,Z_0 (Fig. 1b). The elastic deformations u_i,v_i,w_i of the mass element dm_i located by d_i as measured from the appendage attachment point O_i, are defined with respect to the undeformed coordinate system x_i,y_i,z_i, respectively (Fig. 2). r_c represents a vector locating the instantaneous center of mass O_c with respect to the center of mass O_0 of the undeformed body. The net offset of O_i relative to O_c is given by Π_i, while the geometric offset from O_0 is represented by σ_i. The appendage deployment velocity along the x_i direction is given by U_i.

As can be expected, derivation of the governing equations of motion for such a system undergoing arbitrary librations, deformations, and deployment of appendages is extremely lengthy and time consuming. It is described in detail by Lips.[13] It is sufficient to note here that the governing equations in the librational degrees of freedom are highly nonlinear, nonautonomous, and coupled. Even the simplest of the equations contains more than 70 terms! The problem is further aggravated by the fact that deformations u_i,v_i,w_i appearing in the preceding equations are themselves functions of the librational response.

B. Appendage Dynamics

In the analysis thus far, the physical characteristics of the appendages have been left unspecified. Ultimately, however, to obtain a solution of the librational dynamics, one may identify the type of appendage and solve the associated equations governing flexibility. This can involve even greater effort than the attitude equations themselves.

In the following analysis, the spacecraft is assumed to have beam-type appendages that are representative of antennae, stabilizing booms, and the supporting bars associated with experimental packages and solar arrays. Furthermore, one would expect long trusslike structures to display an overall beam-type behavior. Taking the beam to be of the Euler-Bernoulli type makes it essentially one dimensional; hence, its characteristics are specified by only one spatial variable. Such a "slender" system is assumed to experience simple flexure only; that is, effects of rotary inertia and shear deformation are considered negligible.

Consider the ith appendage to be a beam deploying with local velocity U_i along the x_i direction, where the x_i axis coincides with the undeformed neutral axis of the appendage (Fig. 2). As pointed out earlier, appendage attachment position is offset from the overall system center of mass. The linear density ρ_i, stiffness E_i, and cross-sectional inertia $J_{jk,i}$ are allowed to vary along the length of the boom. Also, the appendage is permitted to have an arbitrary initial orientation in space and is free to undergo transverse as well as axial deformations. Axial foreshortening associated with transverse deformations is allowed for as well.

As in the case of general librational motion, the equations governing the oscillations of the elastic appendages were also found to be nonlinear, nonautonomous, and coupled. Consequently, the overall system is too complex to be amenable to any closed-form solution. This development is also discussed at length by Lips.[13]

Through linearization of the vibration equations and considering the appendage to be uniform, the problem becomes somewhat more tractable. This, together with the assumption of axial rigidity in addition to continuity considerations, results in deployment velocity being uniform along the length of the appendage[15]; that is, $U(x,t) = U(t)$.

The range of application of the general formulation can be appreciated by summarizing its essential features: 1) satellite of arbitrary geometry in a general orbit undergoing three-axis librations; 2) arbitrary number, type, and orientation of flexible appendages deploying independently at arbitrary velocity and acceleration; 3) appendage permitted to have variable mass density, flexural rigidity, and cross-sectional area along its length; 4) governing nonlinear equations account for gravitational effects, shifting center of mass, appendage offset, together with transverse as well as axial oscillations; and 5) modified Eulerian rotations chosen to make the governing equations applicable to both spin-stabilized and gravity-gradient orientations.

Even when linearized, the vibration equations retain their complex character and hence are not amenable to any simple closed-form solution. To obtain some appreciation as to the character of the motion, the vibrations were analyzed using the assumed-mode method.

C. Computational Considerations

Together, the librational and vibrational degrees of freedom form a conjugate system; hence, they must be solved simultaneously. This was achieved with the help of an AMDAHL 470-V6-II digital computer. The numerical integration routine (GEARB-I.M.S.L.) was based on the implicit Adam's method with built-in error control. The integration package required a routine (SYSTM) to define the system dynamics in terms of explicit expressions for the first-order derivative of the state vector.[16]

The general program was set up to accommodate an arbitrary number of assumed modes and six booms, four in the x-y plane and two in the x-z plane. Assuming a two-mode representation results in a system of 54 first-order equations. Modal integration coefficients were determined independently by numerical quadrature. Where possible, these integrals were evaluated analytically as well. Accompanied by a liberal use of comment cards, the program exceeded 3800 lines. However, no storage limitations were encountered; although execution times could not be ignored as CPU values of 50-100 were not uncommon. Particularly time consuming are integrations involving small appendage lengths. To cope with the relatively small step size demanded by the high-frequency oscillations, a two-stage integration procedure was established, thus allowing for one complete change in such parameters during the course of the integration.

III. Results and Discussion

The objective was to develop a model that can treat the transient and steady-state effects of flexibility and deployment in a relatively general manner in order to assess their interactions with the attitude dynamics. This paper illustrates the application of the formulation to two classical satellite configurations. The amount of information generated through a systematic variation of the large number of variables inherent in the problem is rather extensive. For conciseness, only some of the more typical results, useful in establishing trends, are recorded here.

A. Gravity-Stabilized Spacecraft

To treat this case, the general configuration of Fig. 2 is simplified, such that cantilevered to a central rigid body are two diametrically opposed uniform flexible beam-type appendages that can be independently deployed. In the nominal equilibrium condition, the undeformed appendages are aligned along the local vertical ($\boldsymbol{R}_c$) with boom 1 pointing in the outward direction ($\phi_I = 0$) relative to the center of force O_I.

Calculations were carried out for this two-boom, gravity-gradient configuration at an orbital altitude of 6000 km. The physical characteristics of the appendages coincide with those of the RAE antennae ($\rho = 0.023024$ kg/m, $EJ_{33} = 7.85$ Nm^2). The principal mass moment of inertias of the central rigid body, I_{jj}, are taken to be as: $I_{11} = I_{33} = 1535$ kgm^2; $I_{22} = 18$ kgm^2. For clarity, at some places, x, y, z are replaced by subscripts 1, 2, 3, respectively. Three-axis response to differing attitude or boom initial conditions was examined.

Figure 3 compares the response of rigid and flexible satellites with the instantaneous deployment length L varying from 0-100 m. Corresponding performance with the fully deployed appendage length L fixed at 100 m is also included.

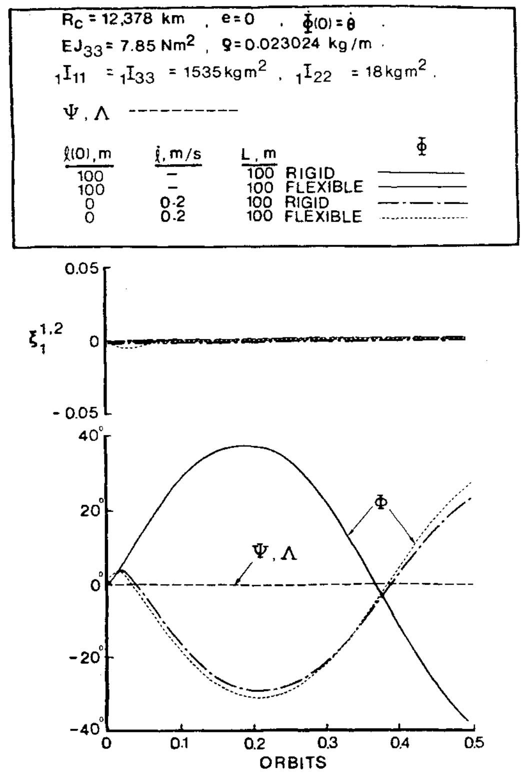

Fig. 3 Three-axis response of a satellite to an impulsive pitch disturbance.

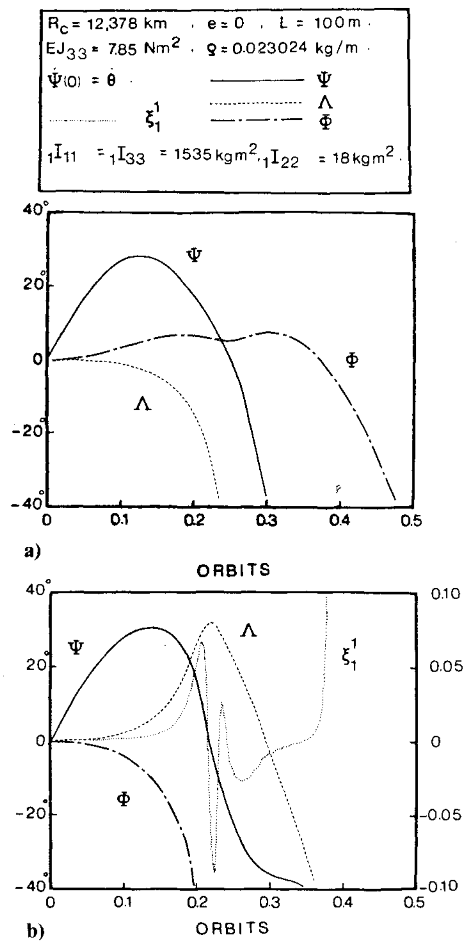

Fig. 4 Three-axis response of a satellite with fully deployed appendages to an impulsive out-of-plane disturbance: a) rigid booms, b) flexible booms.

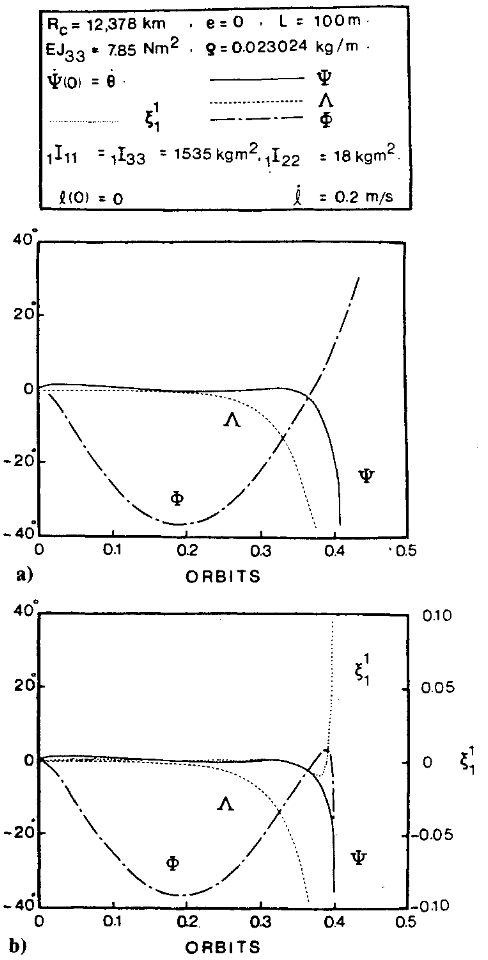

Fig. 5 Effect of boom deployment on the three-axis response of a satellite to an impulsive out-of-plane disturbance: a) rigid booms, b) flexible booms.

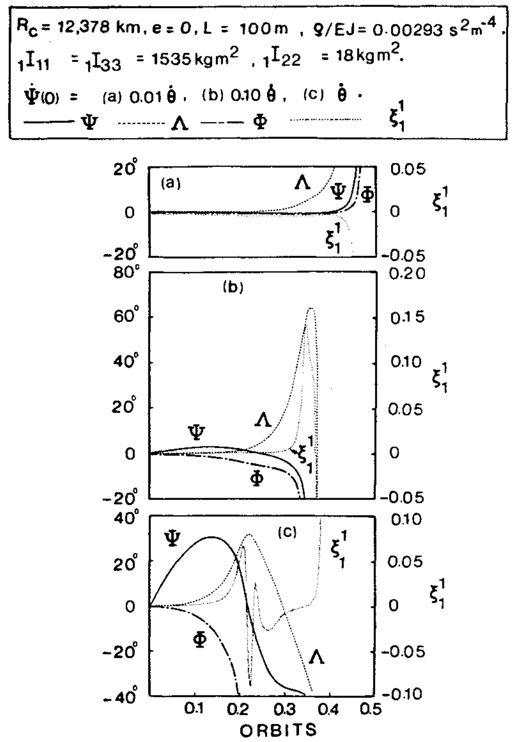

Fig. 6 Effect of magnitude of an impulsive out-of-plane disturbance on three-axis response.

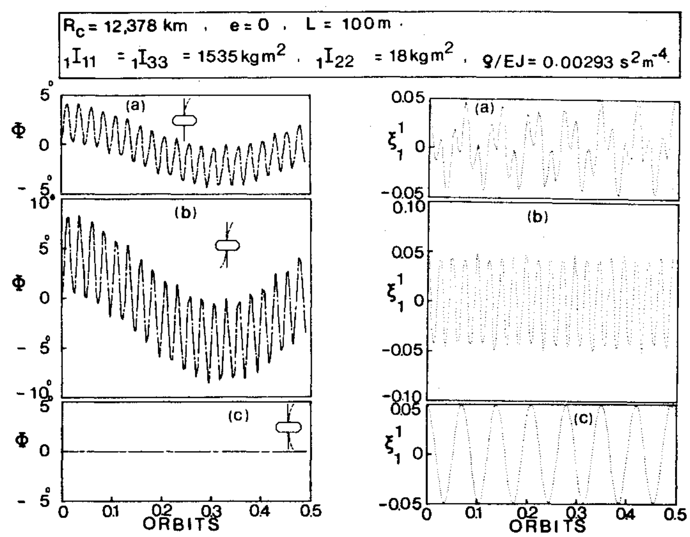

Fig. 7 Planar response of the gravity-gradient configuration to different initial elastic deformations.

In all cases, the system is subjected to the initial impulsive pitch (planar) excitation of $\dot{\Phi}(0)=\dot{\theta}$; however, roll and yaw degrees of freedom are left undisturbed. As can be expected, large-amplitude (35 deg) pitch motion results; however, it is of interest to recognize that there are virtually no coupling effects as roll and yaw motions are essentially absent, so is the vibratory response ($\xi_n^{i,j}$ is the generalized coordinate associated with the nth mode for the ith and jth flexible appendages). Note that for the nondeploying condition, near absence of the flexible appendage vibration results in pitch response that is identical to the rigid case. However, during deployment, slight vibration of the flexible members in the early stage does bring about noticeable difference in the resulting pitch response. The early reversal in amplitude may be explained by the conservation of momentum.

Just how strong coupling effects can be is demonstrated by applying an impulsive initial condition of $\dot{\Psi}(0)=\dot{\theta}$ to the roll degree of freedom only (Fig. 4). Large amplitudes result in both yaw and pitch, as well as for vibrations. In fact, the overall motion becomes unstable within half an orbit. This is in marked contrast to the stable response associated with the planar initial condition of the same magnitude. Note also the significant effect of deployment on the nature of the coupled response (Fig. 5). Large displacements are also experienced in this case within less than 0.5 orbit. Flexibility, however, has minimal effect, except near the point of instability where it alters pitch response quite dramatically.

Having considered two very different types of attitude disturbances, the next logical step was to assess system sensitivity to a given disturbance. To this end the system was subjected to a set of three impulsive roll velocities of increasing magnitude (Fig. 6). Note that the strong coupling effects continue to persist even in the presence of a small disturbance. The larger the roll rate, the earlier the instability sets in. The results also suggest that large displacements in librational and vibrational degrees of freedom are closely related.

Boom response to an initial tip displacement equal to 5% of the length is displayed in Fig. 7 by a plot of the generalized coordinate associated with the first admissible function and the corresponding pitch libration. Symmetric initial displacements of the booms produce no pitching, while antisymmetric initial conditions result in a peak pitch ≈ 8deg. Disturbing only one boom initially yields librations less than 5 deg. Note a considerable difference in frequency between the vibrational response for the symmetric case as opposed to the other two situations. Such high-frequency behavior is eliminated during symmetric oscillation, since pitch itself is not excited.

Figure 8 describes three-dimensional librational response when the tip of the appendage is displaced (in the orbital plane) by an amount equal to 1% of its length. As expected, the case of symmetric appendage disturbance closely resembles the planar response data given earlier in Fig. 7c. There is, however, approximately 1 deg of roll apparent after half an orbit. This is in contrast to the antisymmetric case where both roll and yaw remain unexcited (Fig. 8b). Pitch responds in a manner analogous to that in the planar case, except that now the peak amplitude is only around 1.5 deg. However, a strikingly dramatic effect of coupling is revealed when the system is subjected to a disturbance in the form of tip displacement of one of the booms (Fig. 8a). Initially, up to around one-quarter of an orbit, only a small-amplitude, pitch librational motion is excited. However, subsequently, both yaw and roll appear, grow in magnitude monotonically, and, in turn, cause large-amplitude vibration, driving the system unstable within half an orbit! This is in marked contrast to the apparently stable behavior in the planar case, even with more severe initial conditions as given in Fig. 7a. This emphasizes the significance of coupling effects in a study of the class of spacecraft with flexible appendages.

Although not shown here for reasons of brevity, results were also obtained to assess effects of several other parameters on the dynamics of two-boom, gravity-gradient configuration free to undergo three-axis librations. The use of higher modes to represent appendage vibration showed only a minor difference in amplitude without affecting the general character of the response. Similarly, the effect of shifting center of mass, offset of the appendage attachment, and the appendage foreshortening during transverse oscillations on librational response was found to be negligible (amplitude change less than 5%). More noticeable was the shift in phase, which was also present during deployment of the appendages. Of course, as shown by the authors,[17] deployment can affect the system response substantially and under a certain critical combination of parameters can drive it unstable.

B. Spin-Stabilized Spacecraft

In addition to the gravity-stabilized concept, another equilibrium orientation involves a satellite (or a momentum wheel) spinning at a rate much greater than the orbital rate, with the axis of spin normal to the orbital plane. Using coordinates as defined in Fig. 2, the x,y body-fixed axes lie in the spin plane (orbital plane). This section studies the

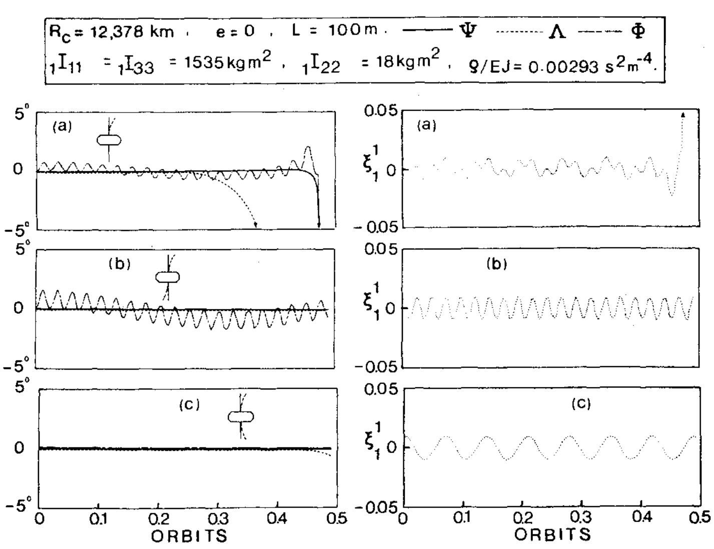

Fig. 8 Three-axis response of the gravity-gradient configuration to different initial elastic deformations.

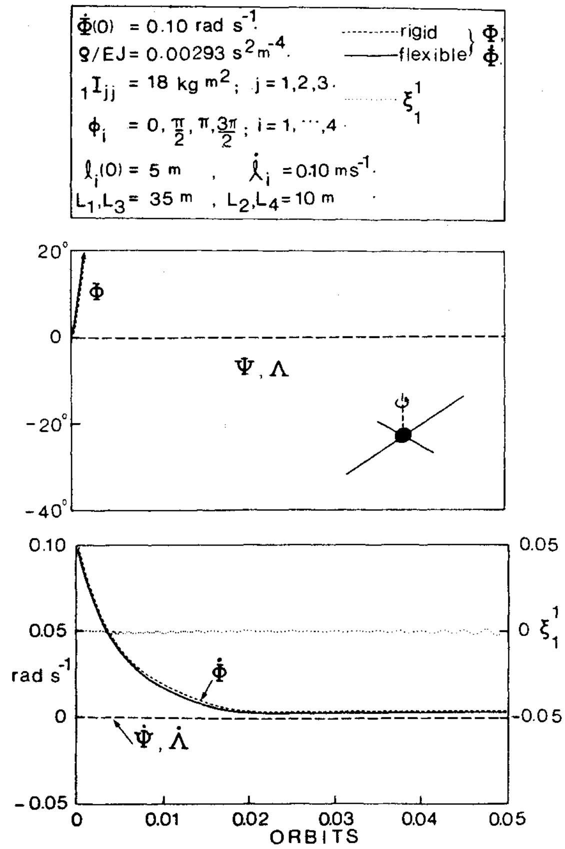

Fig. 9 Three-axis response of a spinning spacecraft during deployment of rigid or flexible appendages.

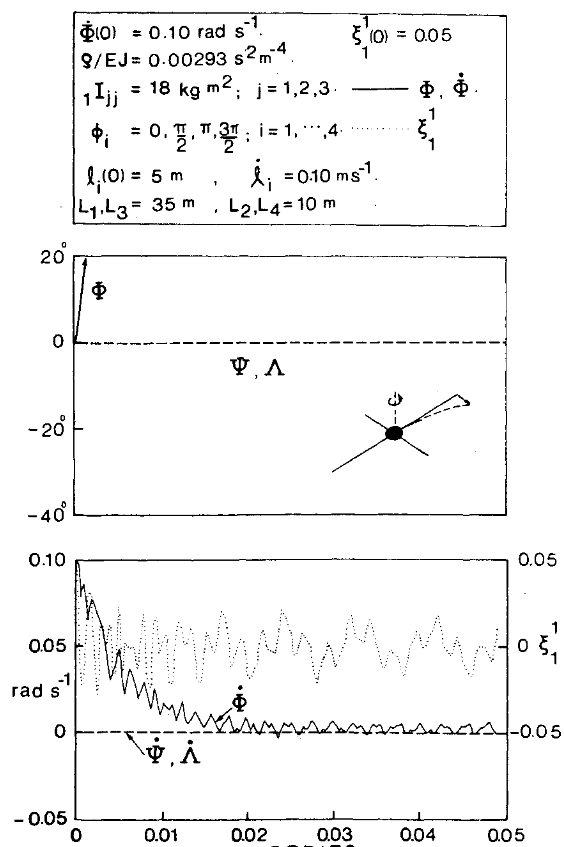

Fig. 10 Three-axis response of a spinning spacecraft during deployment of flexible appendages with one boom initially deformed.

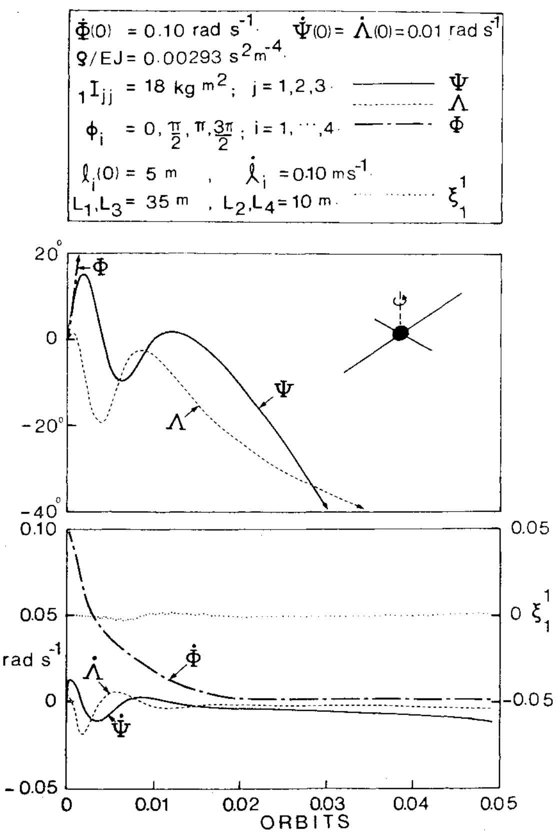

Fig. 11 Three-axis response of a spinning spacecraft with flexible deploying appendages when subjected to out-of-plane attitude disturbances.

dynamics of a system having four mutually orthogonal, flexible, deploying, uniform beam-type appendages numbered 1 through 4 lying in the spin plane. Orbital characteristics, together with boom properties ρ, EJ_{33}, are the same as in the gravity-stabilized case; however, the core inertias are taken as $I_{11} = I_{22} = I_{33} = 18$ kgm^2. The length of each pair of diametrically opposed booms is similar to that of the Allouette II satellite.

Presented in Fig. 9 is the three-axis attitude response of the system (initially spinning at 0.1 rad/s) during deployment of appendages at 0.10 m/s. Although all booms have the same starting length and deploy at the same rate, booms 2 and 4 stop deploying at 10 m, whereas 1 and 3 extend to 35 m. Results for rigid appendages are also included for comparison. Despin of the pitch degree of freedom is according to the conservation of angular momentum. The configuration is highly stable with the pitch rate attaining a constant value following deployment, and there are no out-of-plane librations. Note the effect of flexibility is essentially negligible. This is consistent with the low level of vibrations. In fact, the appendage vibration is virtually absent until the first set of booms stops deploying. Even after 0.05 orbit (685 s) oscillations at the tip stay much less than 1% of the boom length.

Displacing boom 1 [$\phi_1 = 0$, $l_1(0) = 5$ m] by 0.25 m at the tip in the spin plane at the start of deployment still fails to excite any roll/yaw motion (Fig. 10). However, considerable interaction between the pitch and the flexible appendages leads to high-frequency modulation of the pitch rate, a result similar to that observed in the gravity-gradient case.

Figure 11 presents response of the system to an impulsive roll/yaw disturbance equal to 10% of the nominal initial spin rate. Large-amplitude displacements result leading to tumbling motion in less than 11 min. Furthermore, not only the pitch rate but also the yaw rate decreases significantly. On the other hand, the roll rate appears to grow. Note that the strong roll coupling effects experienced in the gravity-gradient case are not dominant here and the appendage oscillations are minimal.

IV. Concluding Remarks

Important features of the study and conclusions based on the results may be summarized as follows.

1) Using a relatively general formulation, the paper examines nonlinear transient response of spacecraft having flexible, deploying, beam-type appendages. Two important configurations of practical importance are considered: gravity gradient and spin stabilized. The ease with which such diverse classes of satellite configurations could be simulated demonstrates the versatility of the general formulation.

2) Through a systematic variation of system parameters, the degree of interaction between flexibility, deployment, and attitude motion is assessed.

3) The analysis suggests that, in general, the parameters such as shifting center of mass, appendage offset, foreshortening, and higher modes, which substantially complicate the formulation, have little effect on the magnitude of the response. Hence, one can neglect them, at least during the preliminary design stage, with considerable saving in computational time and effort.

4) Coupled character of the motion significantly affects the system dynamics; hence, caution should be exercised in utilizing results based on simplified planar analyses.

5) Interaction between flexibility and libration leads to an increase in the frequency of appendage oscillation together with a high-frequency modulation of the attitude response.

6) Stable librations do not excite significant appendage motion, whereas initial boom displacements can result in very noticeable changes in attitude.

7) Librational study suggests that depending on orbital parameters and physical properties of booms, there are critical values of appendage length and deployment rate for which the satellite can tumble over.

8) The small-amplitude oscillations evident both with the gravity-gradient and spin-stabilized response justify a linear vibration analysis.

Acknowledgments

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References

[1] Noll, R.B., Deyst, J.J., and Spenny, C.H., "A Survey of Structural Flexibility Effects on Spacecraft Control Systems," Paper 69-116, Jan. 1969.

[2] Shrivastave, S.K., Tschann, C., and Modi, V.J., "Librational Dynamics of Earth Orbiting Satellites," *Proceedings of the 14th Congress on Theoretical and Applied Mechanics,* Kurukshetra, India, 1969, pp. 284-306.

[3] Likins, P.W., "Dynamics and Control of Flexible Space Vehicles," NASA TR-32-1329, Jan. 1970.

[4] Likins, P.W., "Analytical Dynamics and Non-rigid Spacecraft Simulation," NASA TR-32-1593, July 1974.

[5] Likins, P.W., "Interaction Problems Between the Dynamics and Control System for Nonrigid Spacecraft," *Proceedings of the ESA Symposium on Dynamics and Control of Non-Rigid Spacecraft,* ESA SP 117, Frascati, Italy, May 1976, pp. 265-271.

[6] Likins, P.W., "The Influence on Dynamics and Control Theory of the Spacecraft Attitude Control Problem," Invited Lecture, *Proceedings of the Sixth Canadian Congress of Applied Mechanics,* Vancouver, May-June 1977, Vol. 1, pp. 321-335.

[7] Likins, P.W. and Bouvier, H.K., "Attitude Control of Nonrigid Spacecraft," *Astronautics & Aeronautics,* Vol. 9, May 1971, pp. 64-71.

[8] Modi, V.J., "Attitude Dynamics of Satellites With Flexible Appendages—A Brief Review," *Journal of Spacecraft and Rockets,* Vol. 11, Nov. 1974, pp. 743-751.

[9]Williams, C.J.H., "Dynamics Modelling and Formulation Techniques for Non-Rigid Spacecraft," *Proceedings of the ESA Symposium on Dynamics and Control of Non-Rigid Spacecraft,* ESA SP 117, Frascati, Italy, May 1976, pp. 53-70.

[10]Garg, S.C., Morrow, L.D., and Mamen, R., "Strapdown Navigation Technology: A Literature Survey," *Journal of Guidance and Control,* Vol. 1, May-June 1978, pp. 161-172.

[11]Stuhlinger, E., "First Steps into Space, 1946-1978," *Journal of Spacecraft and Rockets,* Vol. 16, Jan.-Feb. 1979, pp. 3-9.

[12]Roberson, R.E., "Two Decades of Spacecraft Attitude Control," *Journal of Guidance and Control,* Vol. 2, Jan.-Feb. 1979, pp. 3-8.

[13]Lips, K.W., "Dynamics of a Large Class of Satellites with Deploying Flexible Appendages," Ph.D. Dissertation, University of British Columbia, Aug. 1980.

[14]Misra, A.K. and Modi, V.J., "The Influence of Satellite Flexibility on Orbital Motion," *Proceedings of the AIAA Symposium on Large Flexible Satellites,* Va., 1977, pp. 59-74; also, *Celestial Mechanics,* Vol. 17, 1978, pp. 145-165.

[15]Tabarrok, B., Leech, C.M., and Kim, Y.I., "On the Dynamics of an Axially Moving Beam," *Journal of the Franklin Institute,* Vol. 297, March 1974, pp. 201-220.

[16]Conte, S.K., *Elementary Numerical Analysis,* McGraw-Hill, 1965, pp. 204, 250.

[17]Lips, K.W. and Modi, V.J., "General Dynamics of a Large Class of Flexible Satellite Systems," Paper 79-192; presented at the 30th International Aeronautical Federation Conference, Munich, West Germany, Sept. 1979; *Acta Astronautica,* Vol. 7, No. 12, 1980, pp. 1349-1360.